Period 1, April 30, 2025

Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

| $t$ (hours) | 2 | 5 | 9 | 11 | 12 |
|---|---|---|---|---|---|
| $L(t)$ (cars per hour) | 15 | 40 | 24 | 68 | 18 |

The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t-2)(t-5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function $L$. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time $t$ is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \leq t \leq 12$.

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

(d) For $0 \leq t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \leq t \leq 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

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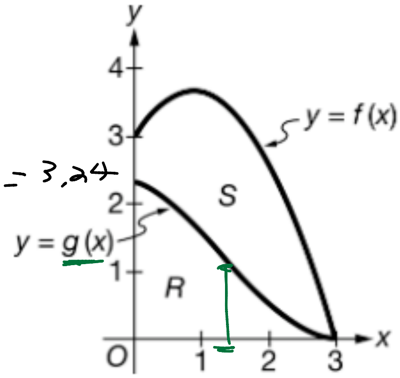
$$5\int_0^6 (30+5(T-2)(T-5)e^{-0.2T})\,dT + 8\int_6^{12}(30+5(T-2)(T-5)e^{-0.2T})\,dT$$

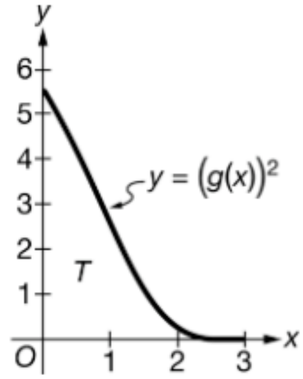
$5(210.14)$ $+8(309.93)$

$1050.7 + 2479.44 = 3530.14$

Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

$\int_0^3 (g(x))dx = 3.24$




The function $f$ is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function $g$ is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of $f$ and $g$ and the regions $R$ and $S$. $R$ is the region bounded by the graph of $g$ and the $x$- and $y$-axes. Region $R$ has area $3.24125$. $S$ is the region bounded by the $y$-axis and the graphs of $f$ and $g$.

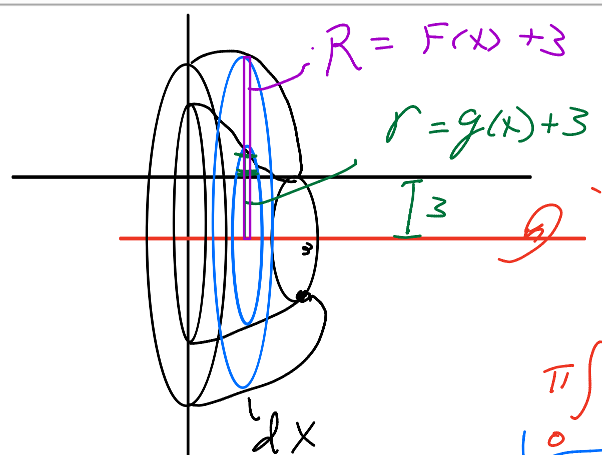
The figure above on the right shows the graph of $y = (g(x))^2$ and the region $T$. $T$ is the region bounded by the graph of $y = (g(x))^2$ and the $x$- and $y$-axes. Region $T$ has area $5.32021$.

$\int_0^3 (g(x)^2)dx = 5.32021$

(b) Find the volume of the solid generated when region $S$ is revolved about the horizontal line $y = -3$.

Volume is 255.0589

3 / 10000 Word Limit




$\pi\int_0^3 (R^2 - r^2)dx$

$\pi\int_0^3 \left[(F(x)+3)^2 - (g(x)+3)^2\right]dx$

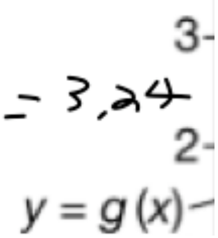
$\underbrace{\pi\int_0^3 [f(x)+3]^2 dx}_{\text{calculator}} - \pi\int_0^3 (g(x)+3)^2 dx$

$$\pi\int_0^3 \left[g(x)^2 + 6(g(x)) + 9\right]dx$$

$$\pi\left[\int_0^3 g(x)^2\,dx + 6\int_0^3 g(x)\,dx + \int_0^3 9\,dx\right]$$

5.32 +

$$\int_0^3 (g(x))\,dx = 3.24$$




Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

Let $f$ be the function defined above.

Slope $F'(x) = -1 + -3\left(\sin\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}$

$F'(3) = -1 - 3\left(\sin\frac{3\pi}{2}\right)\cdot\frac{\pi}{2} = -1 - 3\cdot -1\cdot\frac{\pi}{2} = -1 + \frac{3\pi}{2} = m$

(b) Write an equation for the line tangent to the graph of $f$ at $x = 3$.

Point $F(3) = -3 + 3\cos\frac{3\pi}{2} = -3 + 3\cdot 0 = -3$

Point $(3, -3)$ $m = -1 + \frac{3\pi}{2}$

$y - y_1 = m(x - x_1)$

$y + (+3) = \left(-1 + \frac{3\pi}{2}\right)(x - 3)$

$-3$ $-3$

$y = \left(-1 + \frac{3\pi}{2}\right)(x - 3) - 3$

7 units apart

(c) Find the average value of $f$ on the interval $-3 \le x \le 4$.

height of Rectangle

$$\frac{\left[\int_{-3}^{0} \underbrace{\sqrt{9-x^2}\,dx}_{\frac{9\pi}{4}} + \int_{0}^{4} \underbrace{\left(-x+3\cos\frac{\pi x}{2}\right) dx}_{=-8}\right]}{7}$$

$-\frac{1}{2}(4)^2 + \cancel{\frac{6}{\pi}\sin\frac{4\pi}{2}} + C$

$-\frac{1}{2}x^2 + 3 \cdot \frac{2}{\pi}\sin\frac{\pi x}{2} + C$

$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

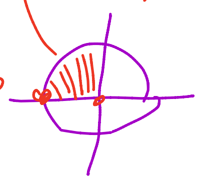
Circle

Center $(0,0)$

Radius $= 3$

$\frac{1}{4}\pi r^2 = \frac{1}{4}\pi 3^2 = \frac{9}{4}\pi$

$-3$



$\int \cos\frac{\pi x}{2}\,dx = \int \cos u \cdot \frac{2}{\pi}\,du$

$u = \frac{\pi x}{2}$

$du = \frac{\pi}{2}\,dx$

$\frac{2}{\pi}\,du = dx$

$\frac{2}{\pi}\int \cos u\,du$

$\frac{2}{\pi}\sin u + C$

$$\boxed{\frac{\frac{9\pi}{4} - 8}{7} =}$$

(d) Must there be a value of $x$ at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

continuous

L + To —

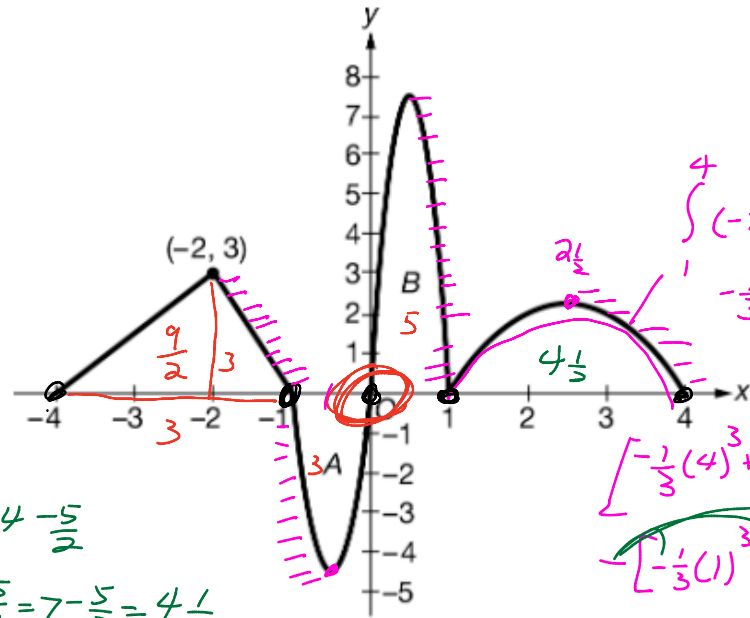
$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{for } -3 \le x \le 0 \Rightarrow F(0)=3 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \rightarrow F(0)=3 \end{cases}$$

$F'(x) = 0$

$F(-3) = 0$

$F(4) = -1$

EVT




Graph of $f$

$\frac{-63}{3}+24+4-\frac{5}{2}$

$-21+28-\frac{5}{2}=7-\frac{5}{2}=4\frac{1}{2}$

$\int_{1}^{4}(-x^2+5x-4)\,dx$

$-\frac{1}{3}x^3+\frac{5}{2}x^2-4x\Big[_{1}^{4}$

$\left[-\frac{1}{3}(4)^3+\frac{5}{2}(4)^2-4(4)\right]$

$-\left[-\frac{1}{3}(1)^3+\frac{5}{2}(1)^2-4(1)\right]$

$-\frac{64}{3}+\cancel{40}-\cancel{16}+\frac{1}{3}-\frac{5}{2}+4=$

The continuous function $f$ is defined for $-4 \le x \le 4$. The graph of $f$, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x=-\frac{1}{2}$, $x=\frac{1}{2}$, and $x=\frac{5}{2}$. It is known that $f(x)=-x^2+5x-4$ for $1 \le x \le 4$. The areas of regions $A$ and $B$ bounded by the graph of $f$ and the $x$-axis are 3 and 5, respectively. Let $g$ be the function defined by $g(x)=\int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

$g(0)=\int_{-4}^{0}F(T)\,dT=\frac{9}{2}-3=\frac{9}{2}-\frac{6}{2}=\frac{3}{2}$

$g(4)=\int_{-4}^{4}F(T)\,dT=\underbrace{\int_{-4}^{0}F(T)\,dT}_{\frac{3}{2}}+\int_{0}^{4}F(T)\,dT$

$5+4\frac{1}{2}$

$1\frac{1}{2}+5+4\frac{1}{2}=11$

negative To Positive

(b) Find the absolute minimum value of $g$ on the closed interval $[-4, 4]$. Justify your answer.

$g'(x) = 0$

$g(-4) = \int_{-4}^{-4} F(T)\,dT = 0$

$g'(x) = F(x) = 0$ or $\emptyset$

$g(-4)$ $g(4)$

| X | Y |
|---|---|
| -4 | 0 |
| 4 | 11 |
| -1 | |
| 0 | $\frac{3}{2}$ |
| 1 | |

$g(0) = \int_{-4}^{0} F(x)\,dx = \frac{3}{2}$

(c) Find all intervals on which the graph of $g$ is concave down. Give a reason for your answer.

$g(x) = \int_{-4}^{x} F(T)\,dT$

$g'(x) = F(x)$

$g''(x) = F'(x) = \text{Slope of } F(x) =$

$g''(x) = -$

when is slope of $F(x)$ negative

$(-2, -\frac{1}{2}) \cup (\frac{1}{2}, 1) \cup (2\frac{1}{2}, 4)$

| $t$ (hours) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $B(t)$ (miles per hour) | 1 | 8 | 1.5 | $-5$ | 11 |

Find a Slope = 2.5
(0,1) (4,11)

Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time $t$ hours is given by a $\boxed{\text{differentiable function}}$ $B$ for $0 \le t \le 4$. Values of $B(t)$ for selected times $t$ are given in the table above. Chloe's velocity, in miles per hour, at time $t$ hours is given by the piecewise function $C$ defined by

$$C(t) = \begin{cases} te^{4-t^2} & \underline{\text{for } 0 \le t \le 2} \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

$C(0) = 0e^{4-0^2} = 0$
$C(1) = 1 \cdot e^{4-1} = e^3$

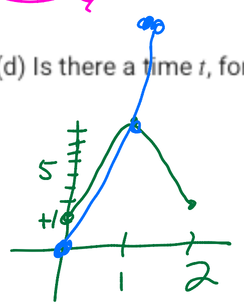
(c) Is there a time $t$, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

MVT

$\frac{11-1}{4-0} = 2.5$

(d) Is there a time $t$, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

Yes




$B(T) = C(T)$

$B(T) - C(T) = 0$

$B(0) - C(0) = 1 - 0 = 1$

$B(1) - C(1) = 8 - e^3 = \text{negative}$

IVT $B(T) - C(T) = 0$

Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

$2(x^2) + 3\left(\frac{2}{3}x\right)^2 - 4x\left(\frac{2}{3}x\right) = 36$

$2x^2 + \frac{4}{3}x^2 - \frac{8}{3}x^2 = 36$

$\frac{6}{3}x^2 - \frac{4}{3}x^2 = 36 \Rightarrow \frac{\cancel{3}2}{\cancel{2}3}x^2 = 36 \cdot \frac{3}{2}$

(a) Show that $\frac{dy}{dx} = \frac{2y-2x}{3y-2x}$. =

(c) Find the positive value of $x$ at which the curve has a vertical tangent line. Show the work that leads to your answer.

$\frac{dy}{dx} = \emptyset = \frac{a}{0}$

$x^2 = 54$

$\frac{2y-2x}{3y-2x}$

$3y - 2x = 0$

$y = \frac{2}{3}x$

$x = \sqrt{54}$

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(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

$\frac{dy}{dx} = \frac{2y-2x}{3y-2x}$

$2(6)^2 + 3(y)^2 - 4 \cdot 6 \cdot y = 36$

$72 + 3y^2 - 24y = 36$

$-36 \qquad -36$

(6,2) $\quad \frac{2(2)-2(6)}{3(2)-2(6)} = \frac{4-12}{6-12} = \frac{-8}{-6} = \frac{4}{3}$

$3y^2 - 24y + 36 = 0$

$3(y^2 - 8y + 12) = 0$

$3(y-2)(y-6) = 0$

$y = 2$ or $6$

(6,6) $\quad \frac{2(6)-2(6)}{3(6)-2(6)} = \frac{12-12}{18-12} = 0$

$\frac{d}{dT}$

(d) Let $x$ and $y$ be functions of time $t$ that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of $x$ is 2, the value of $y$ is $-2$, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

$$2\cdot 2x' \cdot \frac{dx}{dT} + 3\cdot 2y' \cdot \frac{dy}{dT} - \left[4\frac{dx}{dT}\cdot y + 4x\cdot\frac{dy}{dT}\right] = 0$$

$$4(2)\cdot\frac{dx}{dT} + 6\cdot(-2)\cdot 4 - \left[4\cdot\frac{dx}{dT}\cdot -2 + 4\cdot 2\cdot 4\right] = 0$$

$$8\frac{dx}{dT} + -48 + 8\frac{dx}{dT} - 32 = 0$$

$$+48 \quad +32 \quad +48 \quad +32$$

$$\frac{16\frac{dx}{dT}}{16} = \frac{80}{16}$$

$$\frac{dx}{dT} = 5$$

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$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right),$$

$$dT \cdot \frac{dH}{dT} = \frac{\frac{1}{2}(H-1)\cos\frac{T}{2}\cdot dT}{H-1}$$

$H-1$

$u = H-1$
$du = dH$

$$\int \frac{1}{H-1}\,dH = \int \frac{1}{2}\cos\frac{T}{2}\,dT$$

$$\int \frac{1}{2}\cos\frac{T}{2}\,dT$$

$u = \frac{T}{2}$

$du = \frac{1}{2}dT$

$2du = dT$

$\int \frac{1}{2}\cos u \cdot 2\,du$

$\sin u + c$

$$\ln|H-1| + C_1 = \sin\frac{T}{2} + C_2$$

$$-C_1 \qquad -C_1$$

$C_2 - C_1 = C_3$

$$\ln|H-1| = \sin\frac{T}{2} + C_3$$

$$\ln|H-1| = \sin\frac{T}{2} + C$$

$H(0) = 4.$

$T = 0$
$H = 4$

$\ln|4-1| = \sin\frac{0}{2} + c$

$\ln|3| = c$

$\ln 3 = c$

$$\ln_e |H-1| = \left(\sin\frac{\pi}{2} + \ln 3\right) \Leftrightarrow e^{\sin\frac{\pi}{2}+\ln 3} = H-1$$

$$+1 \qquad +1$$

$$e^{\sin\frac{\pi}{2}+\ln 3} + 1 = H$$

$$e^{\sin\frac{\pi}{2}} \cdot e^{\ln 3} + 1 = H$$

$$3e^{\sin\frac{\pi}{2}} + 1 = H \quad \text{MCQ}$$